

# Potentials in Energy Momentum Tensor and the Equation of State

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The energy momentum tensor of the perfect fluid is a simplified but successful model of wide practical use in astrophysics. In the previous researches, some important information such as the self-energy of the interaction of the particles and influence of the gravitational potential are usually ignored or introduced phenomenologically. In this letter, the analysis and calculation show that, the interactive potentials of the particles can be described by introducing an extra potential  $W$ , which acts like negative pressure. For the gases in a star, the state functions are strongly influenced by the gravitational potential, and only the temperature  $T$  is independent variable, but the other state functions such as mass-energy density  $\rho$  and pressure  $P$  can be expressed as simple functions of  $T$ . These relations can provide some new insights into the dynamical behavior and structure of a star.

Key Words: *energy momentum tensor, equation of state, gravitational potential, self-energy of interaction*

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In astrophysics, the energy momentum tensor of the particles in a star is usually described by the Einstein's perfect fluid model

$$T^{\mu\nu} = (\rho + P)U^\mu U^\nu - Pg^{\mu\nu}. \quad (1)$$

Theoretical analysis and experiments all show (1) is a successful approximation. Due to the effort of several researchers such as Israel and Stewart[1, 2], Carter[3] and Lichnerowicz[4], the energy momentum tensor  $T^{\mu\nu}$  has been developed into a treasure box for classical fluid theory, which contains not only energy and momentum, but also heat flux, spatial stress

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and viscosity[5, 6]. Furthermore, the relativistic hydrodynamics with variational principle and Cartan's exterior algebra was elegantly established[7].

However, some important information is ignored in the perfect fluid model (1). One is the self-energy of the internal interactions of the particles such as electromagnetic potential and nonlinear potential of a spinor, the other is influence of the gravity on the motion of particles. In this letter, we discuss the two realistic problems in detail. The discussion shows that the self-energy of interaction can be merged into the mass-energy density, and an extra potential term  $W$  should be introduced into  $T^{\mu\nu}$  which acts as negative pressure. The meaning of mass-energy density  $\rho$  should be redefined to include the contribution of the self-energy. The gases in thermal equilibrium inside a star is a state automatically formed, so the equation of state can not be arbitrarily given, and only the temperature is the independent variable. This property is much helpful to study the interior behave of a star, and leads to an elegant and natural model for stellar structure[8]. Besides, the analysis also shows the consistence and connection among thermodynamics, classical mechanics and field theory.

In conventional thermodynamics, some concepts are usually not explained definitely, e.g. even the temperature  $T$  was not clearly defined for large energy range. This is inconvenient for theoretical analysis and easily leads to confusion. In the case of the monatomic gases, considering that the thermal motion of particles in the comoving coordinate system is fundamental, so we define the temperature  $T$  of the gases as follows

$$kT \equiv \frac{2}{3}\bar{K} = \frac{2}{3}\frac{1}{N}\sum_n^N K_n, \quad (2)$$

where  $k$  is the Boltzmann constant,  $\bar{K}$  the mean shifting kinetic energy of particles in a micro comoving volume element  $V$ ,  $N$  the particle number in  $V$ ,  $K_n$  the kinetic energy of  $n$ -th particle. (2) is the most popular description in the usual textbooks.

Assuming the distribution of kinetic energy of the particles is given by

$$d\mathcal{P} = f\left(\frac{K}{kT}\right) \frac{dK}{kT}, \quad (3)$$

which satisfies

$$\int_0^\infty d\mathcal{P} = 1, \quad \int_0^\infty K d\mathcal{P} = \frac{3}{2}kT, \quad \int_0^\infty K^2 d\mathcal{P} = \frac{3}{2\sigma}(kT)^2, \quad (4)$$

where  $\sigma$  is a constant related to the concrete distribution function for given particles. In usual statistical mechanics, the distribution functions are usually defined in the phase space of

coordinate and momentum of the particles, but such selection is sometimes inconvenient for calculation of state functions, especially in the case of relativistic gases. Since the following discussions have not any explicit relation with distribution function  $f$ , and at most use the second order moment, so the kinetic energy distribution (3) is much convenient and the quantities defined by Eq(4) are enough. The following examples show how  $\sigma$  varies with distribution  $f$ ,

$$d\mathcal{P} = \exp\left(-\frac{K}{kT}\right) \sqrt{\frac{4K}{\pi kT}} \frac{dK}{kT}, \quad \sigma = \frac{2}{5}, \quad (\text{Maxwell distribution}), \quad (5)$$

$$d\mathcal{P} = \left(1 + \frac{40}{177} \exp\left(\frac{K}{kT}\right)\right)^{-1} \frac{dK}{\ln(217/40)kT}, \quad \sigma = \frac{15}{38}, \quad (6)$$

$$d\mathcal{P} = \left(1 + \frac{2K}{3(n-2)kT}\right)^{-n} \frac{2(n-1)dK}{3(n-2)kT}, \quad \sigma = \frac{n-3}{3(n-2)}, \quad (7)$$

$$d\mathcal{P} = \delta\left(\frac{K}{kT} - \frac{3}{2}\right) \frac{dK}{kT}, \quad \sigma = \frac{2}{3}, \quad (\text{the upper bound of } \sigma). \quad (8)$$

For the quantized nonlinear fields such as spinors, in microscopic view, the energy momentum tensor can be expressed by the following classical approximation[9, 10, 11]

$$T^{\mu\nu} = \sum_k (m_k u_k^\mu u_k^\nu + w_k g^{\mu\nu}) \delta^3(\vec{x} - \vec{X}_k) \sqrt{1 - v_k^2}, \quad (9)$$

where  $m_k$  is the proper mass of the  $k$ -th particle,  $w_k$  is the proper potential,  $u_k^\mu$  4-vector velocity,  $v_k$  the speed,  $\vec{X}_k(t)$  the central coordinate. In the case of ideal gas, we have  $w_k = 0, (\forall k)$ . The physical meaning of  $w_k$  can be explained by the following example. Consider the following Lagrangian of self-interaction

$$\mathcal{L}_A = \sum_k \mathcal{L}_{A|k}, \quad \mathcal{L}_{A|k} = \frac{1}{2} (\partial_\mu A_{\nu|k} \partial^\mu A_k^\nu - s^2 A_k^\alpha A_{\alpha|k}) + e_k q_k^\mu A_{k\mu}, \quad (10)$$

where  $A_k^\mu$  stands for the self potential of the  $k$ -th spinor,  $s = 0$  corresponds to the electromagnetic field. Making classical approximation, we generally have[11, 12, 13]

$$\begin{aligned} T^{\mu\nu}|_A &= \sum_k \partial^\mu A_{\alpha|k} \partial^\nu A_k^\alpha - g^{\mu\nu} \mathcal{L}_A \\ &\rightarrow \sum_k [(m'_k + p'_k) u_k^\mu u_k^\nu - p'_k g^{\mu\nu}] \sqrt{1 - v_k^2} \delta^3(\vec{x} - \vec{X}_k), \end{aligned} \quad (11)$$

where  $\vec{X}_k$  is the central coordinate of the particle,  $m'_k$  stands for the proper energy of  $A_k^\mu$  in mean sense, and  $p'_k$  for the internal pressure. By (11), the parameters in the central

coordinate system are calculated by

$$m'_k = - \int_{R^3} \mathcal{L}_{A|k} d\bar{x} = \frac{1}{2} e_k \int_{R^3} q_k^\mu A_{k\mu} d^3\bar{x} > 0, \quad (12)$$

$$p'_k = \frac{1}{3} \left( m'_k + \int_{R^3} (\bar{\nabla} A_k^\alpha \bar{\nabla} A_{\alpha|k} + 4\mathcal{L}_{A|k}) d^3\bar{x} \right) = -\frac{1}{3} \left( m'_k + \int_{R^3} s^2 A_k^\alpha A_{\alpha|k} d^3\bar{x} \right), \quad (13)$$

in which we have used  $\bar{\partial}_0 A^\alpha = 0$  and  $-\Delta A_k^\alpha + s^2 A_k^\alpha = e q_k^\alpha$  in the central coordinate system. From (13) we learn  $p'_k < 0$ . So different from the photons which have  $p_{\text{ph}} = \frac{1}{3} \rho_{\text{ph}}$ , the static field  $A_k^\alpha$  provides negative pressure. By (12) and (13), we find (9) is of general meaning, in which  $m_k$  can be explained as the total mechanical mass of the  $k$ -th particle including the self-energy of interaction,  $p'_k < 0$  is the total potential acting as negative pressure, which leads to  $w_k > 0$  in (9).

In order to calculate the statistical expectation value of Eq(9), denote

$$\tilde{P} \equiv -\frac{1}{3}(T_x^x + T_y^y + T_z^z), \quad K_k = \frac{m_k}{\sqrt{1-v_k^2}} - m_k, \quad (14)$$

then we have

$$T_0^0 = \sum_k \left( K_k + m_k + w_k \sqrt{1-v_k^2} \right) \delta^3(\vec{x} - \vec{X}_k), \quad (15)$$

$$\tilde{P} = \sum_k \frac{1}{3} \left( K_k + m_k - (m_k + 3w_k) \sqrt{1-v_k^2} \right) \delta^3(\vec{x} - \vec{X}_k). \quad (16)$$

In Eq(15) and (16), we can directly solve the mean value of  $K_k, m_k$  as the function of  $kT$  and  $\bar{m}$  by (4), but the rest terms include the factor  $\sqrt{1-v_k^2}$ , which is difficult for estimation. We resolve the problem by the following trick, which naturally includes the influence of the gravitational potential.

To research the thermodynamical properties of gas, we use piston and cylinder to drive gas. In astrophysics, we have more ideal piston and cylinder, that is the spacetime with Friedmann-Robertson-Walker(FRW) metric, which is absolutely adiabatic and reversible. The FRW metric drives the gases homogeneously expanding and contracting as the scale factor  $a$  varies, and the results have generally meanings according to the principle of equivalence.

The line element of the FRW spacetime is given by

$$ds^2 = dt^2 - a^2(t) (dr^2 + h^2(r) d\theta^2 + h^2(r) \sin^2 \theta d\varphi^2), \quad (17)$$

where the specific form of  $h(r)$  is not important. Solving the geodesic we have the following relations[10, 14, 15]

$$v_k = \frac{b_k}{\sqrt{a^2 + b_k^2}}, \quad \sqrt{1 - v_k^2} = \frac{a}{\sqrt{a^2 + b_k^2}}. \quad (18)$$

$$a = \frac{\sigma \bar{m} b}{\sqrt{kT(kT + 2\sigma \bar{m})}}, \quad kT = \frac{\sigma \bar{m} b^2}{a(a + \sqrt{a^2 + b^2})}, \quad (19)$$

where  $b_k, b$  are determined by initial velocity. Since we firstly solve the micro motion of particles and then solving the expectation value of a variable, the results should be the same as that we firstly do statistical average for the variable and then solve its macro changing value, so we have the following calculation. By (18), we have relation

$$\sqrt{1 - v_k^2} = \frac{d}{da} \frac{a}{\sqrt{1 - v_k^2}} = \frac{1}{m_k} \frac{d}{da} [a(K_k + m_k)]. \quad (20)$$

Then by (19), we get

$$\frac{1}{N} \sum_k m_k \int \sqrt{1 - v_k^2} d\mathcal{P} = \frac{d}{da} \left( \frac{a}{N} \sum_k \int (K_k + m_k) d\mathcal{P} \right) = \bar{m} - \frac{3\sigma \bar{m} kT}{2(\sigma \bar{m} + kT)}, \quad (21)$$

$$\frac{1}{N} \sum_k w_k \int \sqrt{1 - v_k^2} d\mathcal{P} = \frac{d}{da} \left( \frac{a}{N} \sum_k \frac{w_k}{m_k} \int (K_k + m_k) d\mathcal{P} \right) = \bar{w} - \frac{3\sigma \bar{\mu} \bar{m} kT}{2(\sigma \bar{m} + kT)}, \quad (22)$$

where the mean parameters are defined by

$$\bar{w} = \frac{1}{N} \sum_k w_k, \quad \bar{\mu} = \frac{1}{N} \sum_k \frac{w_k}{m_k}. \quad (23)$$

In the case  $w_k > 0$ , by Eq(22) we have

$$\sum_k w_k \int \sqrt{1 - v_k^2} d\mathcal{P} > 0, \quad \sigma \bar{\mu} < \frac{2\bar{w}}{3\bar{m}}. \quad (24)$$

For the same kind particles, by  $\bar{w} = \bar{\mu} \bar{m}$ , we get  $\sigma < \frac{2}{3}$ .

Similar to thermodynamical laws have not relation with piston and cylinder, Eq(21) and (22) also have not any relation with the ‘piston’  $a(t)$ , so they are independent of any dynamical process, and are generally valid for ideal gases in the comoving coordinate system.

We define the mass-energy density  $\rho$  and pressure  $P$  of the particles in comoving coordinate system as usual[9]

$$\rho \equiv \sum_k \frac{m_k}{\sqrt{1 - v_k^2}} \delta^3(\vec{x} - \vec{X}_k), \quad P \equiv \frac{1}{3} \sum_k \frac{m_k v_k^2}{\sqrt{1 - v_k^2}} \delta^3(\vec{x} - \vec{X}_k), \quad (25)$$

Then, in mean sense we get

$$\rho = \frac{1}{V} \int d\mathcal{P} \int_V \sum_k (K_k + m_k) \delta^3(\vec{x} - \vec{X}_k) dV = \varrho \left( 1 + \frac{3kT}{2\bar{m}} \right), \quad (26)$$

where  $\varrho = \frac{N\bar{m}}{V}$  is the static mass density. By Eq(15), (16) and (21), (22), we find the expectation value of Eq(9) should take the following form

$$T^{\mu\nu} = (\rho + P)U^\mu U^\nu + (W - P)g^{\mu\nu}, \quad (27)$$

where  $U^\mu$  is the mean velocity satisfying  $U_\mu U^\mu = 1$ , and  $W$  is an ignored term corresponding to potentials  $w_k$ , which acts as negative pressure.

Then according to (21) and (22), Eq(26) and (27) give

$$W = \frac{1}{V} \int d\mathcal{P} \int_V T^0_0 dV - \rho = \varrho \left( \frac{\bar{w}}{\bar{m}} - \frac{3\bar{\mu}\sigma kT}{2(\sigma\bar{m} + kT)} \right), \quad (28)$$

$$P = \frac{1}{V} \int d\mathcal{P} \int_V \tilde{P} dV + W = \varrho \frac{kT}{\bar{m}} \left( 1 - \frac{kT}{2(\sigma\bar{m} + kT)} \right). \quad (29)$$

In case of ideal gases or spinors, Eq(29) can be also derived according to the principle of action[14]. In the ultra-relativistic case  $kT \gg \bar{m}$ , we have  $P \rightarrow \frac{1}{3}\rho$ , which is obviously reasonable in physics.

$W$  can be explained as potentials. If  $w_k = 0$ , we get  $W = 0$ , and the other results reduce to the relations for ideal gas[14, 15]. So the introduction of  $W$  provides an entry to deal with the self-energy of internal interaction potentials of the particles.

The equations (26), (28) and (29) are generally valid relations for any ideal gases in local equilibrium. By (19), noticing  $\varrho = \frac{N\bar{m}}{V}$  and  $V \propto a^3 \propto [kT(kT + 2\sigma\bar{m})]^{-\frac{3}{2}}$ , we get the following state functions in dimensionless form

$$\varrho = \varrho_0 [J(J + 2\sigma)]^{\frac{3}{2}}, \quad (30)$$

$$\rho = \varrho \left( 1 + \frac{3}{2}J \right) = \varrho_0 [J(J + 2\sigma)]^{\frac{3}{2}} \left( 1 + \frac{3}{2}J \right), \quad (31)$$

$$P = \varrho \frac{J(J + 2\sigma)}{2(J + \sigma)} = \varrho_0 [J(J + 2\sigma)]^{\frac{5}{2}} \frac{1}{2(J + \sigma)}, \quad (32)$$

$$W = \varrho \left( \mu - \frac{3\bar{\mu}\sigma J}{2(J + \sigma)} \right) = \varrho_0 [J(J + 2\sigma)]^{\frac{3}{2}} \left( \mu - \frac{3\bar{\mu}\sigma J}{2(J + \sigma)} \right), \quad (33)$$

where  $\varrho_0$  is a constant depending on parameters  $(\bar{m}, b, \sigma, \mu, \bar{\mu})$ , and

$$J = \frac{kT}{\bar{m}}, \quad \mu = \frac{\bar{w}}{\bar{m}}. \quad (34)$$

Surprisingly, under the constraint of gravity, among the state functions only the temperature is independent.

Equations (31)-(33) are based on the assumption that the particles move along geodesic. But the analysis in [11] shows that the central trajectory of a spinor with self interactions, such as nonlinear potential and electromagnetic field, will slightly depart from the exact geodesic. So in strict sense, we should solve the exact trajectories to replace the equations of geodesics (18) and (19). This is a complicated problem. However, in the case  $0 \leq w_k \ll m_k$ , the above results are good approximations to high precision. Moreover these results can be modified by the following treatments.

In (31)-(33), we treat density  $\varrho$  or equivalently the volume  $V$  as an independent state function, and then derive the function  $\varrho(J)$  according to the continuity equation. In the comoving coordinate system with the following Gaussian type metric[9]

$$g_{\mu\nu} = \text{diag}(g_{00}, -\tilde{g}_{ab}), \quad (a, b) \in \{1, 2, 3\}, \quad (35)$$

where  $\tilde{g}_{ab}$  is the spatial metric, for energy-momentum tensor (27), we check the following Gibbs-Duhem's law

$$\delta Q = d[(\rho + W)V] + (P - W)dV, \quad (36)$$

where  $\delta Q$  denotes the heat received by the  $N$  particles, and

$$V = \sqrt{\tilde{g}}\Delta x\Delta y\Delta z, \quad (\tilde{g} = \det(\tilde{g}_{ab})) \quad (37)$$

is the micro spatial volume occupied by  $N$  given particles.

We trace the motion of these particles. In the adiabatic process, we have  $\delta Q = 0$ . Then by  $U_\mu T^{\mu\nu}_{;\nu} = 0$ , we get the continuity equation for Eq(27) as

$$U^\mu \partial_\mu (\rho + W) + (\rho + P)U^\mu_{;\mu} = 0. \quad (38)$$

Denoting the proper time by  $d\tau$ , then  $\frac{d}{d\tau} = U^\mu \partial_\mu$ , Eq(38) becomes

$$\begin{aligned} 0 &= \sqrt{|g|} \frac{d}{d\tau} (\rho + W) + (\rho + P) \left( \sqrt{|g|} \partial_\mu U^\mu + \frac{d}{d\tau} \sqrt{|g|} \right), \\ &= \sqrt{g_{00}} \left( \frac{d}{d\tau} \left( (\rho + W) \sqrt{\tilde{g}} \right) + (P - W) \frac{d}{d\tau} \sqrt{\tilde{g}} \right) \\ &\quad + (\rho + P) \sqrt{\tilde{g}} \left( \sqrt{g_{00}} \partial_\mu U^\mu + \frac{d}{d\tau} \sqrt{g_{00}} \right), \end{aligned} \quad (39)$$

where  $g = \det(g_{\mu\nu}) = g_{00}\tilde{g}$ . In the comoving system, we have  $U^\mu = (\sqrt{g^{00}}, 0, 0, 0)$  and  $d\tau = \sqrt{g_{00}}dt$ , then we get

$$\sqrt{g_{00}}\partial_\mu U^\mu + \frac{d}{d\tau}\sqrt{g_{00}} = \sqrt{g_{00}}\partial_t \frac{1}{\sqrt{g_{00}}} + \frac{1}{\sqrt{g_{00}}}\partial_t \sqrt{g_{00}} = 0. \quad (40)$$

Multiplying Eq(39) by the comoving volume element  $\Delta x \Delta y \Delta z$  for the  $N$  given particles, (39) gives (36) in the case  $\delta Q = 0$ .

One may oppugn the generality of Eq(36) due to the collision of particles. As shown in [14], Eq(36) holds in mean sense for any process with elastic collisions, which can be understood as relay race of particles. (36) include the gravitational potential of the particles, which is different from but consistent with the energy-momentum conservation law  $T^{\mu\nu}_{;\nu} = 0$ . For example,  $T^{\mu\nu}_{;\nu} = 0$  holds for a little solid ball, in which the molecules are all fixed at definite position by electromagnetic interaction, and then the influence of the gravity on the thermal movement vanishes. However, in astrophysics the gravity is dominant inside a star, where the gravity drives the particles moving rapidly, and such motion should be exchanged and balanced with thermal motion of the particles. This is the underlying and subtle relation between gravity and equation of state, which is ignored before. If the gravitational potential is taken into account, we get elegant and singularity-free models for the stellar structure[8].

In the case of  $\delta Q \neq 0$ , (36) provides an entry to deal with the radiation and heat produced by nuclear reaction inside a star. Obviously,  $\delta Q$  takes the form  $\delta Q = (\varrho q(J) - E_{\text{radiation}})d\tau$ . Then in the equilibrium process, we also have  $\delta Q = 0$ .

Now we derive the relation  $\varrho(J)$  from (36) for the equilibrium process with  $\delta Q = 0$ . For clearness, we use the static mass density  $\varrho = \frac{N\bar{m}}{V}$  to replace  $V$ . Substituting Eq(26), (28) and (29) into Eq(36), we get dimensionless differential equation

$$\frac{1}{\varrho} \frac{d\varrho}{dJ} = \frac{3[(J + \sigma)^2 - \bar{\mu}\sigma^2]}{(J + \sigma)[J^2 + BJ - 2\mu\sigma]}, \quad (41)$$

and the solution

$$\varrho = \varrho_0(J + \sigma)^{\frac{3\bar{\mu}}{3\bar{\mu}+1}} (J^2 + JB - 2\sigma\mu)^{\frac{3}{2}(1-\frac{\bar{\mu}}{3\bar{\mu}+1})} \left( \frac{2J + B - A}{2J + B + A} \right)^{\frac{\alpha}{2A}}, \quad (42)$$

where parameters are defined by

$$B = (2 + 3\bar{\mu})\sigma - 2\mu, \quad A = \sqrt{8\sigma\mu + B^2}, \quad \alpha = 3(1 + 4\bar{\mu})[2\mu - 3\bar{\mu}\sigma]. \quad (43)$$

(42) shows how the internal potentials  $w_k$  influence the mass density. (42) reduces to (31) if  $\mu = \bar{\mu} = 0$ .



In summary, the above calculations work out all relations in the energy-momentum tensor of the monatomic particles, which include the classical approximation for the self-energy of interaction and the influence of the gravitational potential. Although the realistic process inside a star is complicated, the above treatment includes the dominant parts of these effects. Noticing the effectiveness of the assumptions, the results will provide some new insights into the interior behavior and structure of the stars. In addition, we list some interesting properties of the state functions derived above, which show the profound relationship between the empirical laws and the theoretical concepts:

1. In the case of ideal gas with  $\bar{w} = 0$  at low temperature, (31)-(32) gives the equation of state for the adiabatic monatomic gas

$$P \doteq \rho J \doteq P_0 \rho^{\frac{5}{3}}, \quad (J \ll \sigma, \text{ or } kT \ll \bar{m}), \quad (44)$$

which is identical to the empirical law in thermodynamics. When  $J \gg 1$ , we have  $P \rightarrow \frac{1}{3}\rho$ , so the adiabatic index is not a constant for large range of temperature due to the relativistic effect. These results show the validity of (31)-(33) and the consistence with normal thermodynamics.

2. By (31), letting  $J \rightarrow \infty$  or  $\bar{m} \rightarrow 0$ , we get the Stefan-Boltzmann's law  $\rho \propto T^4$ . This means that the above results automatically include photons, and the Stefan-Boltzmann's law is also valid for the ultra-relativistic particles.
3. In general relativity, all processes occur automatically, and  $\varrho_0$  is independent of any practical process, so it is a constant representing the strength of gravity constraining free particles. Of course,  $\varrho_0$  is related to the property of particles. Furthermore, equation of state (31)-(32) provides a self consistent theory for stellar structure in thermal equilibrium[8].
4. In the case  $\bar{w} > 0$ , the motion of the particles with internal interaction will slightly deviate from the geodesic[11], so consequently the above relations have a very tiny violation. By (42) we find,  $\varrho = 0$  leads to  $J = \frac{2\sigma\mu}{B} \approx \mu$ , which means the zero temperature can not reach. This result may be caused by the deviation of motion from geodesic Eq(18).

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